

PATHFINDER CLASSES

IIT-JAM

2017 – IIT JAM (MATHS)

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By Rajneesh Kumar Srivastava : 9350583318, 01145053318

1. $\int_0^1 \int_x^1 \sin(y^2) dy dx =$

(A) $\frac{1 + \cos 1}{2}$

(B) $1 - \cos 1$

(C) $1 + \cos 1$

(D) $\frac{1 - \cos 1}{2}$

Topic : Double and Triple Integral (Advance Integral Calculus)

Ans.(D)

2. If $f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ (1-x)(px+q) & \text{if } x \geq 0 \end{cases}$

satisfies the assumption of Rolle's theorem in the interval $[-1, 1]$, then the ordered pair (p, q) is

(A) $(2, -1)$

(B) $(2, -1)$

(C) $(-2, 1)$

(D) $(2, 1)$

Topic : Rolle's Theorem

Ans.(D)

3. Let $\phi : \mathbf{R} \rightarrow \mathbf{R}$ be a differentiable function such that ϕ' is strictly increasing with $\phi'(1) = 0$. Let α and β denote the minimum and maximum values of $\phi(x)$ on the interval $[2, 3]$, respectively. Then which one of the following is TRUE ?

(A) $\beta = \phi(3)$

(B) $\alpha = \phi(2.5)$

(C) $\beta = \phi(2.5)$

(D) $\alpha = \phi(3)$

Topic : Maxima Minima of One Variable

Ans.(A)

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4. Let $f_1(x)$, $f_2(x)$, $g_1(x)$, $g_2(x)$ be differentiable functions on $\mathbf{R}$. Let $F(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$ be the determinant of the matrix $\begin{bmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{bmatrix}$. Then $F'(x)$ is equal to

- (A) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2(x) \end{vmatrix}$ (B) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$
(C) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} - \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$ (D) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$

Topic : Differential Calculus

Ans.(B)

5. The number of generators of the additive group $\mathbf{Z}_{36}$ is equal to

- (A) 6 (B) 12
(C) 8 (D) 36

Topic : Group Theory

Ans.(B)

6. $\lim_{n \rightarrow \infty} \frac{\pi}{n} \sum_{k=1}^n \sin\left(\frac{\pi}{2} + \frac{5\pi}{2} \cdot \frac{k}{n}\right) =$

- (A) $\frac{2\pi}{5}$ (B) $\frac{5}{2}$
(C) $\frac{2}{5}$ (D) $\frac{5\pi}{2}$

Topic : Definite Integral

Ans.(C)

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7. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be a twice differentiable function. If $g(u, v) = f(u^2 - v^2)$, then

$$\frac{\partial^2 g}{\partial u^2} + \frac{\partial^2 g}{\partial v^2} =$$

- (A) $4(u^2 - v^2)f''(u^2 - v^2)$
(B) $4(u^2 + v^2)f''(u^2 - v^2)$
(C) $2f'(u^2 - v^2) + 4(u^2 - v^2)f''(u^2 - v^2)$
(D) $2(u - v)^2 f''(u^2 - v^2)$

Topic : Differential Calculus

Ans.(B)

8. Consider the function $f(x, y) = 5 - 4\sin x + y^2$ for $0 < x < 2\pi$ and $y \in \mathbf{R}$. The set of critical points of $f(x, y)$ consists of

- (A) a point of local maximum and a point of local minimum
(B) a point of local maximum and a saddle point
(C) a point of local maximum, a point of local minimum and a saddle point
(D) a point of local minimum and a saddle point

Topic : Maxima and Minima of Two Variables (Advance Differential Calculus)

Ans.(D)

9. Let $f(x) = \frac{x + |x|(1+x)}{x} \sin\left(\frac{1}{x}\right)$, $x \neq 0$

Write $L = \lim_{x \rightarrow 0^-} f(x)$ and $R = \lim_{x \rightarrow 0^+} f(x)$. Then which one of the following is TRUE

- (A) L exists but R does not exist
(B) L does not exist but R exists
(C) Both L and R exist
(D) Neither L nor R exists

Topic : Limit (Applied Analysis)

Ans.(A)

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10. If $\lim_{T \rightarrow \infty} \int_0^T e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then $\lim_{T \rightarrow \infty} \int_0^T x^2 e^{-x^2} dx =$

(A) $\frac{\sqrt{\pi}}{4}$

(B) $\frac{\sqrt{\pi}}{2}$

(C) $\sqrt{2\pi}$

(D) $2\sqrt{\pi}$

Topic : Beta and Gamma Function

Ans. (A)

11. Let $f : \mathbf{R} \rightarrow [0, \infty)$ be a continuous function. Then which one of the following is NOT TRUE?

(A) There exists $x \in \mathbf{R}$ such that $f(x) = \frac{f(0) + f(1)}{2}$

(B) There exists $x \in \mathbf{R}$ such that $f(x) = \sqrt{f(-1)f(1)}$

(C) There exists $x \in \mathbf{R}$ such that $f(x) = \int_{-1}^1 f(t) dt$

(D) There exists $x \in \mathbf{R}$ such that $f(x) = \int_0^1 f(t) dt$

Topic : Applied Analysis

Ans.(C)

12. The interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{1}{(-3)^{n+2}} \frac{(4x-12)^n}{n^2+1}$$

is

(A) $\frac{10}{4} \leq x < \frac{14}{4}$

(B) $\frac{9}{4} \leq x < \frac{15}{4}$

(C) $\frac{10}{4} \leq x \leq \frac{14}{4}$

(D) $\frac{9}{4} \leq x \leq \frac{15}{4}$

Topic : Power Series

Ans.(D)

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13. A particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} = e^{2x} \sin x$$

is

(A) $\frac{e^{2x}}{10} (3 \cos x - 2 \sin x)$

(B) $-\frac{e^{2x}}{10} (3 \cos x - 2 \sin x)$

(C) $-\frac{e^{2x}}{5} (2 \cos x + \sin x)$

(D) $\frac{e^{2x}}{5} (2 \cos x - \sin x)$

Topic : Differential Equation

Ans.(A)

14. $\sum_{n=1}^{\infty} \tan^{-1} \frac{2}{n^2} =$

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$

(D) π

Topic : Definite Integral

Ans.(A)

15. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be a differentiable function such that $f(2) = 2$ and

$$|f(x) - f(y)| \leq 5(|x - y|)^{3/2}$$

for all $x \in \mathbf{R}, y \in \mathbf{R}$. Let $g(x) = x^3 f(x)$. Then $g'(2) =$

(A) 5

(B) $\frac{15}{2}$

(C) 12

(D) 24

Topic : Differentiability (Advance Differential Calculus)

Ans.(D)

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16. Let S be an infinite subset of $\mathbf{R}$ such that $S \setminus \{\alpha\}$ is compact for some $\alpha \in S$. Then which one of the following TRUE ?

- (A) S is a connected set
- (B) S contains no limit points
- (C) S is a union of open intervals
- (D) Every sequence in S has a subsequence converging to an element in S

Topic : Pure Analysis (Compactness)

Ans.(D)

17. Let $\vec{F} = (3 + 2xy)\mathbf{i} + (x^2 - 3y^2)\mathbf{j}$ and let L be the curve

$$\vec{r}(t) = e^t \sin t \mathbf{i} + e^t \cos t \mathbf{j}, \quad 0 \leq t \leq \pi$$

Then

$$\int_L \vec{F} \cdot d\vec{r} =$$

- (A) $e^{-3\pi} + 1$
- (B) $e^{-6\pi} + 2$
- (C) $e^{6\pi} + 2$
- (D) $e^{3\pi} + 1$

Topic : Vector Calculus

Ans.(D)

18. Let P_3 denote the real vector space of all polynomials with real coefficients of degree at most 3. Consider the map $T : P_3 \rightarrow P_3$ given by $T(p(x)) = p''(x) + p(x)$. Then

- (A) T is neither one-one nor onto
- (B) T is both one-one and onto
- (C) T is one-one but not onto
- (D) T is onto but not one-one

Topic : Linear Algebra

Ans.(B)

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19. Which one of the following is TRUE

- (A) Every sequence that has a convergent subsequence is a Cauchy sequence
- (B) Every sequence that has a convergent subsequence is a bounded sequence
- (C) The sequence $\{\sin n\}$ has a convergent subsequence
- (D) The sequence $\left\{n \cos \frac{1}{n}\right\}$ has a convergent subsequence

Topic : Sequence

Ans.(C)

20. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{3} + \sqrt{6}} + \frac{1}{\sqrt{6} + \sqrt{9}} + \dots + \frac{1}{\sqrt{3} + \sqrt{3n+3}} \right) =$

- (A) $1 + \sqrt{3}$
- (B) $\sqrt{3}$
- (C) $\frac{1}{\sqrt{3}}$
- (D) $\frac{1}{1 + \sqrt{3}}$

Topic : Definite Integral

Ans.(A)

21. Let $M = \begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 0 & 1 \end{bmatrix}$ and $x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Then $\lim_{n \rightarrow \infty} M^n x$

- (A) does not exist
- (B) is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- (C) is $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$
- (D) is $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Topic : Matrices

Ans.(C)

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22. The area of the surface $z = \frac{xy}{3}$ intercepted by the cylinder $x^2 + y^2 \leq 16$ lies in the interval

(A) $(20\pi, 22\pi]$

(B) $(22\pi, 24\pi]$

(C) $(24\pi, 26\pi]$

(D) $(26\pi, 28\pi]$

Topic : Double and Triple Integral (Advance Integral Calculus)

Ans.(A)

23. The flux of the vector field

$$\vec{F} = \left(2\pi x + \frac{2x^2 y^2}{\pi} \right) \mathbf{i} + \left(2\pi xy - \frac{4y}{\pi} \right) \mathbf{j}$$

along the outward normal, across the ellipse $x^2 + 16y^2 = 4$ is equal to

(A) $4\pi^2 - 2$

(B) $2\pi^2 - 4$

(C) $\pi^2 - 2$

(D) 2π

Topic : Vector Calculus (Advance Integral Calculus)

Ans.(A)

24. The line integral of the vector field $\vec{F} = zx\mathbf{i} + xy\mathbf{j} + yz\mathbf{k}$ along the boundary of the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, oriented anticlockwise, when viewed from the point $(2, 2, 2)$ is

(A) $-\frac{1}{2}$

(B) -2

(C) $\frac{1}{2}$

(D) 2

Topic : Vector Calculus (Advance Integral Calculus)

Ans.(C)

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25. Let $f(x, y) = \frac{x^2 y}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$. Then

- (A) $\frac{\partial f}{\partial x}$ and f are bounded (B) $\frac{\partial f}{\partial x}$ is bounded and f is unbounded
(C) $\frac{\partial f}{\partial x}$ is unbounded and f is bounded (D) $\frac{\partial f}{\partial x}$ and f are unbounded

Topic : Function of Two Variable (Advance Differential Calculus)

Ans.(B)

26. Let $0 < a_1 < b_1$. For $n \geq 1$, define

$$a_{n+1} = \sqrt{a_n b_n} \text{ and } b_{n+1} = \frac{a_n + b_n}{2}$$

Then which one of the following is NOT TRUE ?

- (A) Both $\{a_n\}$ and $\{b_n\}$ converges, but the limits are not equal
(B) Both $\{a_n\}$ and $\{b_n\}$ converges and the limits are equal
(C) $\{b_n\}$ is decreasing sequence
(D) $\{a_n\}$ is increasing sequence

Topic : Sequence

Ans.(A)

27. Let M be the set of all invertible 5×5 matrices with entries 0 and 1. For each $M \in M$, let $n_1(M)$ and $n_0(M)$ denote the number of 1's and 0's in M , respectively. Then

$$\lim_{M \in M} |n_1(M) - n_0(M)| =$$

- (A) 1 (B) 3
(C) 5 (D) 15

Topic : Matrices

Ans.(A)

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28. For $a > 0$, $b > 0$, let $\vec{F} = \frac{x\vec{i} + y\vec{j}}{b^2x^2 + a^2y^2}$ be a planar vector field. Let

$$C = \{(x, y) \in \mathbf{R}^2 \mid x^2 + y^2 = a^2 + b^2\}$$

be the circle oriented anti-clockwise. Then

$$\oint_C \vec{F} \cdot d\vec{r} =$$

(A) $\frac{2\pi}{ab}$

(B) 2π

(C) $2\pi ab$

(D) 0

Topic : Vector Calculus (Advance Integral Calculus)

Ans.(A)

29. The flux $\vec{F} = y\vec{i} - x\vec{j} + z^2\vec{k}$ along the outward normal, across the surface of the solid

$$\{(x, y, z) \in \mathbf{R}^3 \mid 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq \sqrt{2 - x^2 - y^2}\}$$

is equal to

(A) $\frac{2}{3}$

(B) $\frac{5}{3}$

(C) $\frac{8}{3}$

(D) $\frac{4}{3}$

Topic : Double and Triple Integral (Advance Integral Calculus)

Ans.(D)

30. Let $y(x)$ be the solution of the differential equation

$$(xy + y + e^{-x})dx + (x + e^{-x})dy = 0$$

satisfying $y(0) = 1$. Then $y(-1)$ is equal to

(A) $\frac{e}{e-1}$

(B) $\frac{2e}{e-1}$

(C) $\frac{e}{1-e}$

(D) 0

Topic : Differential Equation

Ans.(B)

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Section – B

1. Let $k, l \in \mathbf{R}$ be such that every solution of

$$\frac{d^2y}{dx^2} + 2k \frac{dy}{dx} + ly = 0$$

satisfies $\lim_{x \rightarrow \infty} y(x) = 0$. Then

(A) $3k^2 + l < 0$ and $k > 0$

(B) $k^2 + l > 0$ and $k < 0$

(C) $k^2 - l \leq 0$ and $k > 0$

(D) $k^2 - l > 0$, $k > 0$ and $l > 0$

Topic : Differential Equation

Ans.(C,D)

2. Let $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = (y-1)(y-3)$$

satisfying the condition $y(0) = 2$. Then which of the following is/are TRUE

(A) The function $y(x)$ is not bounded above

(B) The function $y(x)$ is bounded

(C) $\lim_{x \rightarrow +\infty} y(x) = 1$

(D) $\lim_{x \rightarrow -\infty} y(x) = 3$

Topic : Differential Equation

Ans.(B,C,D)

3. Let M be an $n \times n$ matrix with real entries such that $M^3 = I$. Suppose that $Mv \neq v$ for any non-zero vector v . Then which of the following statements is/are TRUE ?

(A) M has real eigenvalues

(B) $M + M^{-1}$ has real eigenvalues

(C) n is divisible by 2

(D) n is divisible by 3

Topic : Matrices

Ans.(B,C)

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4. Let S be the set of all rational numbers in $(0, 1)$. Then, which of the following statements is/are TRUE ?

(A) S is a closed subset of $\mathbf{R}$

(B) S is not a closed subset of $\mathbf{R}$

(C) S is an open subset of $\mathbf{R}$

(D) Every $x \in (0,1) \setminus S$ is a limit point of S

Topic : Pure Analysis

Ans.(B,D)

5. If X and Y are $n \times n$ matrices with real entries, then which of the following is/are TRUE ?

(A) If $P^{-1}XP$ is diagonal for some real invertible matrix P , then there exists a basis for $\mathbf{R}^n$ consisting of eigenvectors of X

(B) If X is diagonal with distinct diagonal entries and $XY = YX$, then Y is not diagonal

(C) If X^2 is diagonal, then X is diagonal

(D) If X is diagonal and $XY = YX$ for all Y , then $X = \lambda I$ for some $\lambda \in \mathbf{R}$

Topic : Matrices

Ans.(A,B,D)

6. Let G be a group of order 20 in which the conjugacy classes have sizes 1, 4, 5, 5. Then which of the following is/are TRUE ?

(A) G contains a normal subgroup of order 5

(B) G contains a non-normal subgroup of order 5

(C) G contains a subgroup of order 10

(D) G contains a normal subgroup of order 4

Topic : Group Theory

Ans. (A,C)

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7. Let $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ be a function. Then which of the following statements is/are TRUE?
- (A) If f is differentiable at $(0,0)$, then all directional derivatives of f exist at $(0,0)$
- (B) If all directional derivatives of f exist at $(0,0)$, then f is differentiable at $(0,0)$
- (C) If all directional derivatives of f exist at $(0,0)$, then f is continuous at $(0,0)$
- (D) If the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and are continuous in a disc centered at $(0,0)$, then f is differentiable at $(0,0)$

Topic : Function of Two Variables (Advance Differential Calculus)

Ans.(A,D)

8. The volume of the solid

$$\left\{ (x, y, z) \in \mathbf{R}^3 \mid 1 \leq x \leq 2, 0 \leq y \leq \frac{2}{x}, 0 \leq z \leq x \right\}$$

is expressible as

(A) $\int_1^2 \int_0^{2/x} \int_0^x dz dy dx$

(B) $\int_1^2 \int_0^x \int_0^{2/x} dy dz dx$

(C) $\int_1^2 \int_0^z \int_0^{2/x} dy dx dz$

(D) $\int_0^2 \int_{\max[z,1]}^2 \int_0^{2/x} dy dx dz$

Topic : Double and Triple Integral (Advance Integral Calculus)

Ans.(A,B,D)

9. For $\alpha, \beta \in \mathbf{R}$, define the map $\phi_{\alpha, \beta} : \mathbf{R} \rightarrow \mathbf{R}$ by $\phi_{\alpha, \beta}(x) = \alpha x + \beta$. Let $G = \{\phi_{\alpha, \beta} \mid (\alpha, \beta) \in \mathbf{R}^2\}$. For $f, g \in G$, define $g \circ f \in G$ by $(g \circ f)(x) = g(f(x))$. Then which of the following statements is/are TRUE ?
- (A) The binary operation $\circ$ is associative
- (B) The binary operation $\circ$ is commutative
- (C) For every $(\alpha, \beta) \in \mathbf{R}^2$, $\alpha \neq 0$ there exists $(a, b) \in \mathbf{R}^2$ such that $\phi_{\alpha, \beta} \circ \phi_{a, b} = \phi_{1, 0}$
- (D) $(G, \circ)$ is a group

Topic : Group Theory

Ans.(A,C)

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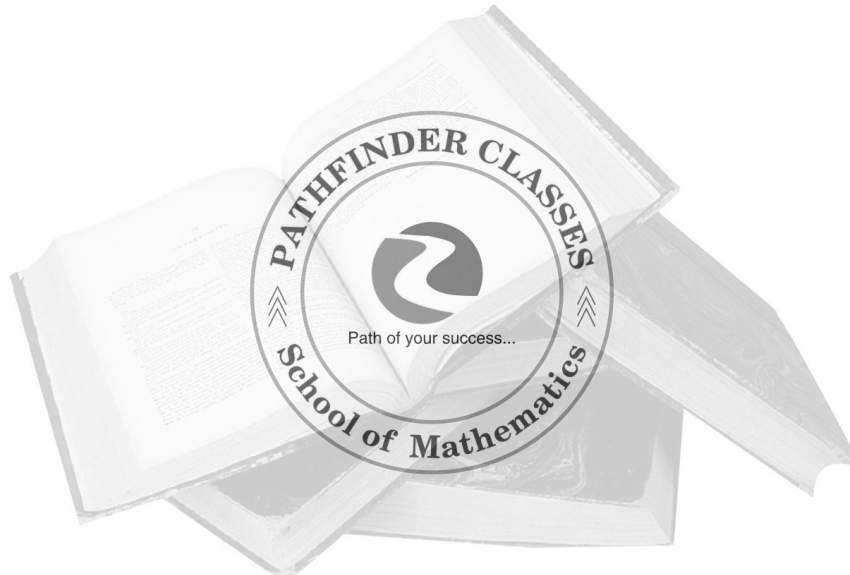
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By Rajneesh Kumar Srivastava : 9350583318, 01145053318

10. Let $\{x_n\}$ be a real sequence such that $7x_{n+1} = x_n^3 + 6$ for $n \geq 1$. Then which of the following statements is/are TRUE ?
- (A) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 1
- (B) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 2
- (C) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to 1
- (D) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to -3

Topic : Sequence

Ans.(A,C)



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Section – C

1. Let P be the point on the surface $z = \sqrt{x^2 + y^2}$ closest to the point (4,2,0). Then the square of the distance between the origin and P is

Topic : Maxima and Minima of One Variable (Advance Differential Calculus)

Ans. 10

2.
$$\frac{1}{2\pi} \left(\frac{\pi^3}{1!3} - \frac{\pi^5}{3!5} + \frac{\pi^7}{5!7} - \dots + \frac{(-1)^{n-1} \pi^{2n+1}}{(2n-1)!(2n+1)} + \dots \right) =$$

Topic : Series

Ans. 0.5

3. Let P be a 7×7 matrix of rank 4 with real entries. Let $a \in \mathbf{R}^7$ be a column vector. Then the rank of $P + aa^T$ is at least

Topic : Matrices

Ans. 4

4. Let G be a subgroup of $GL_2(\mathbf{R})$ generates by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$. Then, the order of G is

Topic : Group Theory

Ans. 6

5.
$$\left(\int_0^1 x^4 (1-x)^5 dx \right)^{-1} =$$

Topic : Beta – Gamma Function

Ans. 1260

6. If the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 = c_1$, $c_1 > 0$, are given by $y = c_2 x^a$, $c_2 \in \mathbf{R}$, then $\alpha =$

Topic : Differential Equation

Ans. 2

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7. For $x > 0$, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x . Then

$$\lim_{x \rightarrow 0^+} x \left(\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \dots + \left\lfloor \frac{10}{x} \right\rfloor \right) =$$

Topic : Applied Analysis

Ans. 55

8. Consider the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 3 & 7 & 8 & 6 & 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 3 & 1 & 7 & 6 & 8 & 2 \end{pmatrix}$ in

S_8 . The number of $\eta \in S_8$ such that $\bar{\eta}\sigma\eta = \tau$ is equal to

Topic : Group Theory

Ans. 0

9. The number of subgroups of $\mathbb{Z}_7 \times \mathbb{Z}_7$ of order 7 is

Topic : Group Theory

Ans. 8

10. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Let M be the matrix whose columns are $v_1, v_2, 2v_1 - v_2, v_1 + 2v_2$ in

that order. Then the number of linearly independent solution of the homogeneous system of linear equation $Mx = 0$ is

Topic : Matrices

Ans. 2

11. Let $a_n = \sqrt{n}$, $n \geq 1$, and $s_n = a_1 + a_2 + \dots + a_n$. Then

$$\lim_{n \rightarrow \infty} \left(\frac{a_n / s_n}{-\ln(1 - a_n / s_n)} \right) =$$

Topic : Sequence

Ans. 1

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12. Let T be the smallest positive real number such that the tangent to the helix

$$\cos t \mathbf{i} + \sin t \mathbf{j} + \frac{t}{\sqrt{2}} \mathbf{k}$$

at $t = T$ is orthogonal to the tangent at $t = 0$. Then the line integral of $\vec{F} = x\mathbf{j} - y\mathbf{i}$ along the section of the helix from $t = 0$ to $t = T$ is

Topic : Double and Triple Integral (Advance Integral Calculus)

Ans. 1.57

13. Let $\alpha, \beta, \gamma, \delta$ be the eigenvalues of the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Then, $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 =$

Topic : Matrices

Ans. 6

14. Let $f(x) = \frac{\sin nx}{\pi \sin x}$, $x \in (0, \pi)$, and let $x_0 \in (0, \pi)$ be such that $f'(x_0) = 0$. Then

$$(f(x_0))^2 (1 + (\pi^2 - 1) \sin^2 x_0) =$$

Topic : Leibnitz Rule

Ans. 1

15. If $y(x) = \int_{\sqrt{x}}^x \frac{e^t}{t} dt$, $x > 0$

then $y'(1) =$

Topic : Leibnitz Rule

Ans. 1.359

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16. For $x > 1$, let

$$f(x) = \int_1^x \left(\sqrt{\log t} - \frac{1}{2} \log \sqrt{t} \right) dt$$

The number of tangents to the curve $y = f(x)$ parallel to the line $x + y = 0$ is

Topic : Leibnitz Rule

Ans. 2

17. The maximum order of a permutations σ in the symmetric group S_{10} is

Topic : Group Theory

Ans. 30

18. For a real number x , define $\lceil x \rceil$ to be the smallest integer greater than or equal to x . Then

$$\int_0^1 \int_0^1 \int_0^1 (\lceil x \rceil + \lceil y \rceil + \lceil z \rceil) dx dy dz =$$

Topic : Double and Triple Integral (Advance Integral Calculus)

Ans. 3

19. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} n! x^{n^2}$$

is

Topic : Power Series

Ans. 0

20. Let $y(x)$, $x > 0$ be the solution of the differential equation

$$x^2 \frac{d^2 y}{dx^2} + 5x \frac{dy}{dx} + 4y = 0$$

satisfying the conditions $y(1) = 1$ and $y'(1) = 0$. Then the value of $e^2 y(e)$ is

Topic : Differential Equation

Ans. 3

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Solution of some Doubtful Problems

Section – B

1. Let $k, l \in \mathbf{R}$ be such that every solution of

$$\frac{d^2y}{dx^2} + 2k \frac{dy}{dx} + ly = 0$$

satisfies $\lim_{x \rightarrow \infty} y(x) = 0$. Then

(A) $3k^2 + l < 0$ and $k > 0$

(B) $k^2 + l > 0$ and $k < 0$

(C) $k^2 - l \leq 0$ and $k > 0$

(D) $k^2 - l > 0, k > 0$ and $l > 0$

Topic : Differential Equation

Ans.(C,D)

Solution. The Auxiliary Equation is

$$m^2 + 2km + l = 0$$

$$\Rightarrow m = \frac{-2k \pm \sqrt{4k^2 - 4l}}{2}$$

$$= -k \pm \sqrt{k^2 - l}$$

$$\therefore y = c_1 e^{(-k + \sqrt{k^2 - l})x} + c_2 e^{(-k - \sqrt{k^2 - l})x}$$

$$= e^{-kx} \left(c_1 e^{(\sqrt{k^2 - l})x} + c_2 e^{-(\sqrt{k^2 - l})x} \right)$$

$$\therefore y(x) \rightarrow 0 \text{ if } x \rightarrow \infty \text{ provided } k^2 - l > 0, k > 0, l > 0$$

$$\text{or } k^2 - l \leq 0, k > 0$$

Section-C

3. Let P be a 7×7 matrix of rank 4 with real entries. Let $a \in \mathbf{R}^7$ be a column vector. Then the rank of $P + aa^T$ is at least

Topic : Matrices

Ans. 4

Solution. Since $a \in \mathbf{R}^7$ be a column vector.

$\therefore a$ is a 7×1 matrix

$\therefore a^T$ is a 1×7 matrix

$\therefore aa^T$ is 7×7 matrix

Since, p is a 7×7 matrix of rank 4 with real entries

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$\therefore$ the rank of $p + aa^T$ is atleast 4.

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Topic : Group Theory

Ans. 6

Solution. Order of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is 2

Order of $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$ is 3

$\therefore$ Order of G which is generated by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$ = $\text{lcm}(2, 3) = 6$

Section–C

19. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} n! x^{n^2}$$

is

Topic : Power Series

Ans. 0

Solution. Radius of convergence $R = \frac{1}{\sup |a_n|^{1/n}}$

$$\sum_{n=0}^{\infty} n! x^{n^2} = 1 + x + 2x^4 + 6x^9 + \dots$$

$$\sup |a_n|^{1/n} = \infty$$

$$\therefore R = \frac{1}{\infty} = 0.$$

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